Tues 11/19

$$\int 3e^{3x} = 3\cdot\frac{1}{3}e^{3x}$$

③ $$\int_0^{\ln(2)}\left(\int_0^1 y e^{xy}\,dx\right)dy$$

$$= \int_0^{\ln(2)} \left(y\cdot\frac{1}{y}e^{xy}\right)\Big|_{x=0}^{1} dy$$

$$= \int_0^{\ln(2)} \left(e^{y} - e^{0}\right)dy = \int_0^{\ln(2)}\left(e^{y}-1\right)dy$$

$$= e^{y} - y\Big|_0^{\ln(2)} = \left(e^{\ln(2)} - \ln(2)\right) - \left(e^{0} - 0\right) = 2 - \ln(2) - 1$$

$$1-\ln(2)$$

Cont

Ex

tha

and

Continued from 11/7

Ex: Find the volume of the solid E that lies between the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=z$.

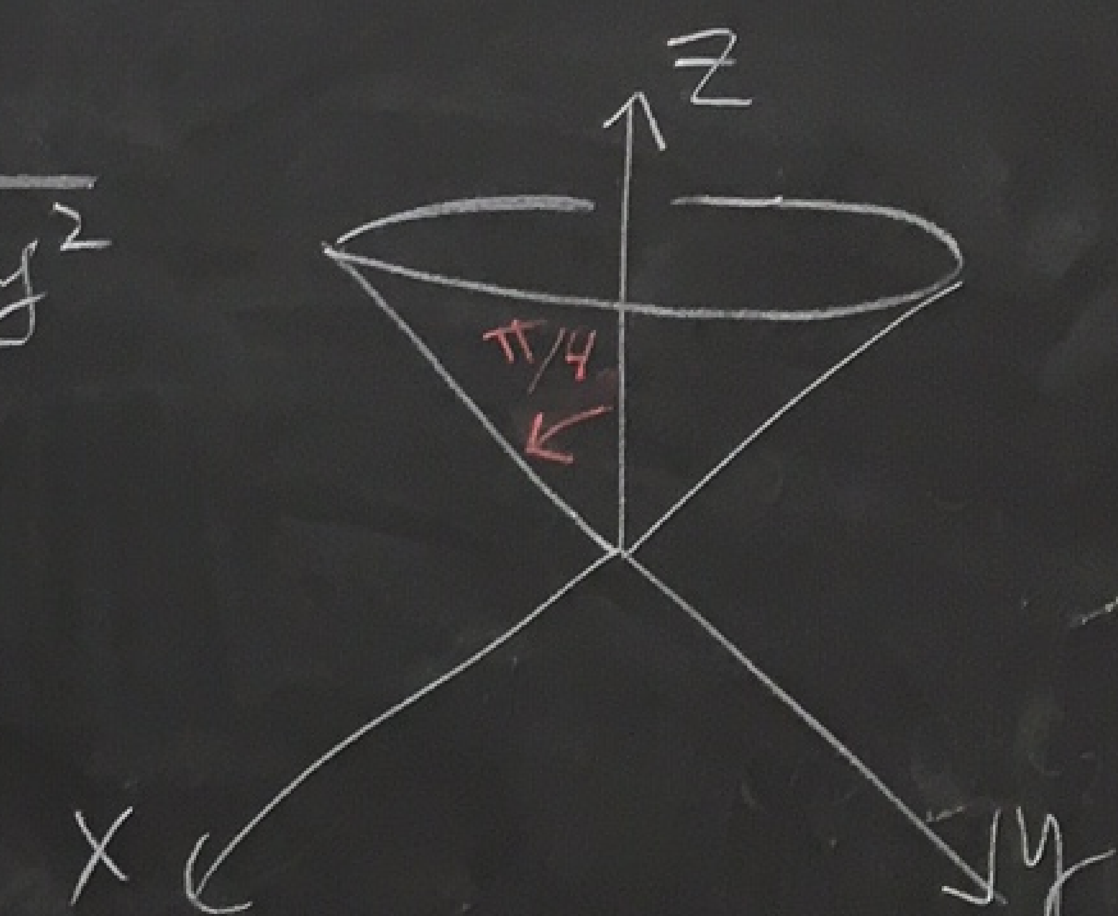
$$x^2+y^2+z^2=z$$

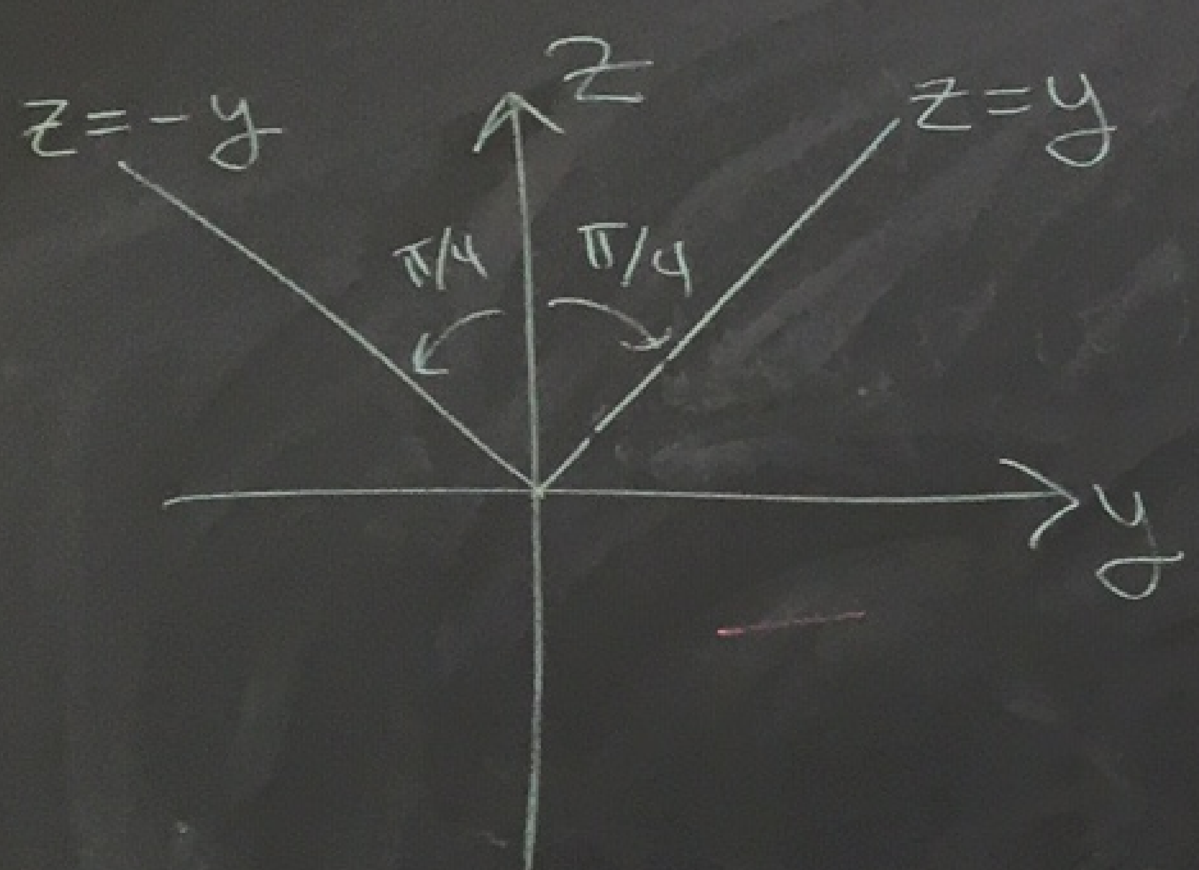
$$(x-0)^2+(y-0)^2+\left(z-\frac{1}{2}\right)^2=\frac{1}{4}=\left(\frac{1}{2}\right)^2$$

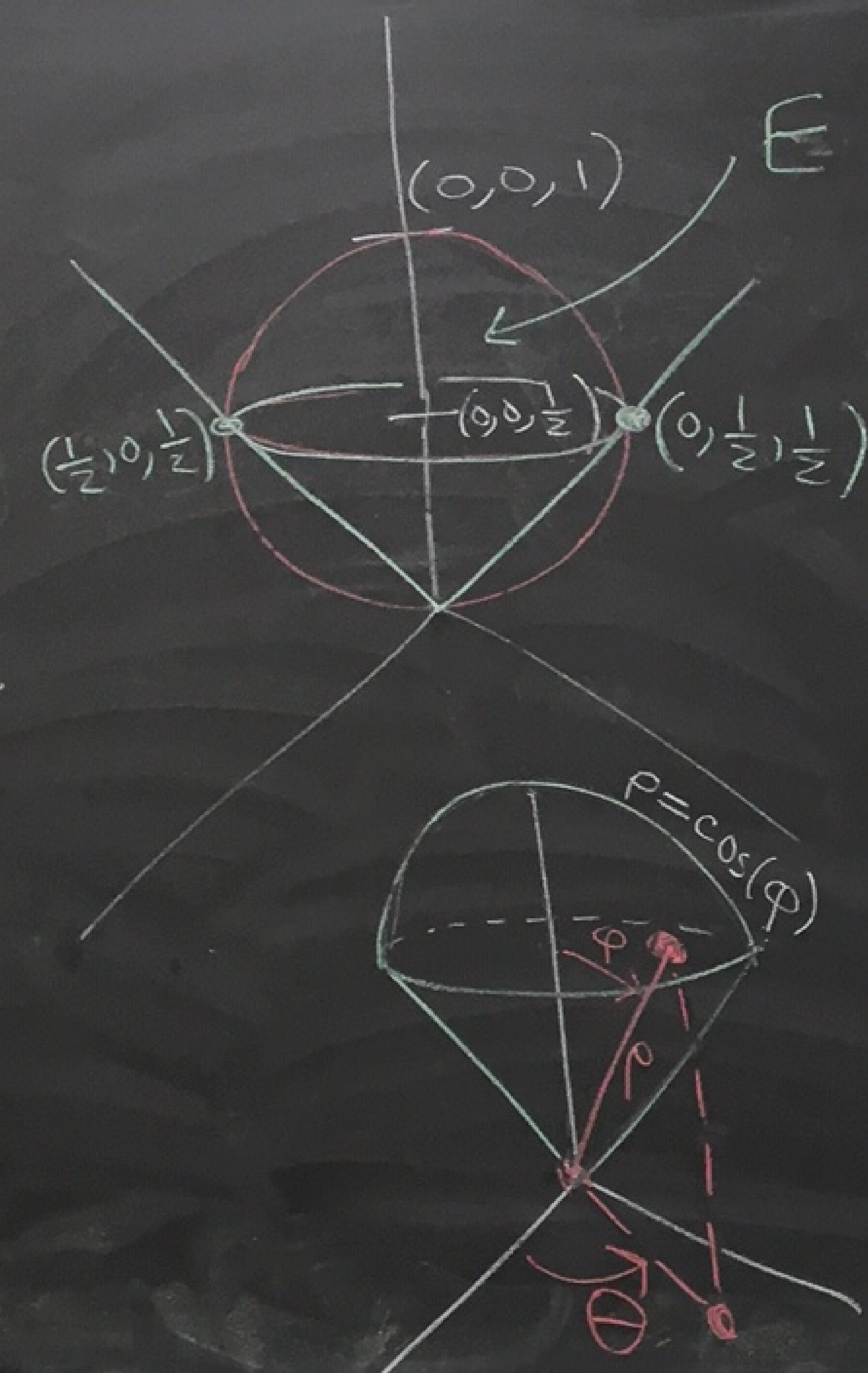
sphere centered at $\left(0,0,\frac{1}{2}\right)$
with radius $\frac{1}{2}$

$$z=\sqrt{x^2+y^2}$$

$$\varphi=\frac{\pi}{4}$$







$$x^2+y^2+z^2=z$$

$$\rho^2=\rho\cos(\varphi)$$

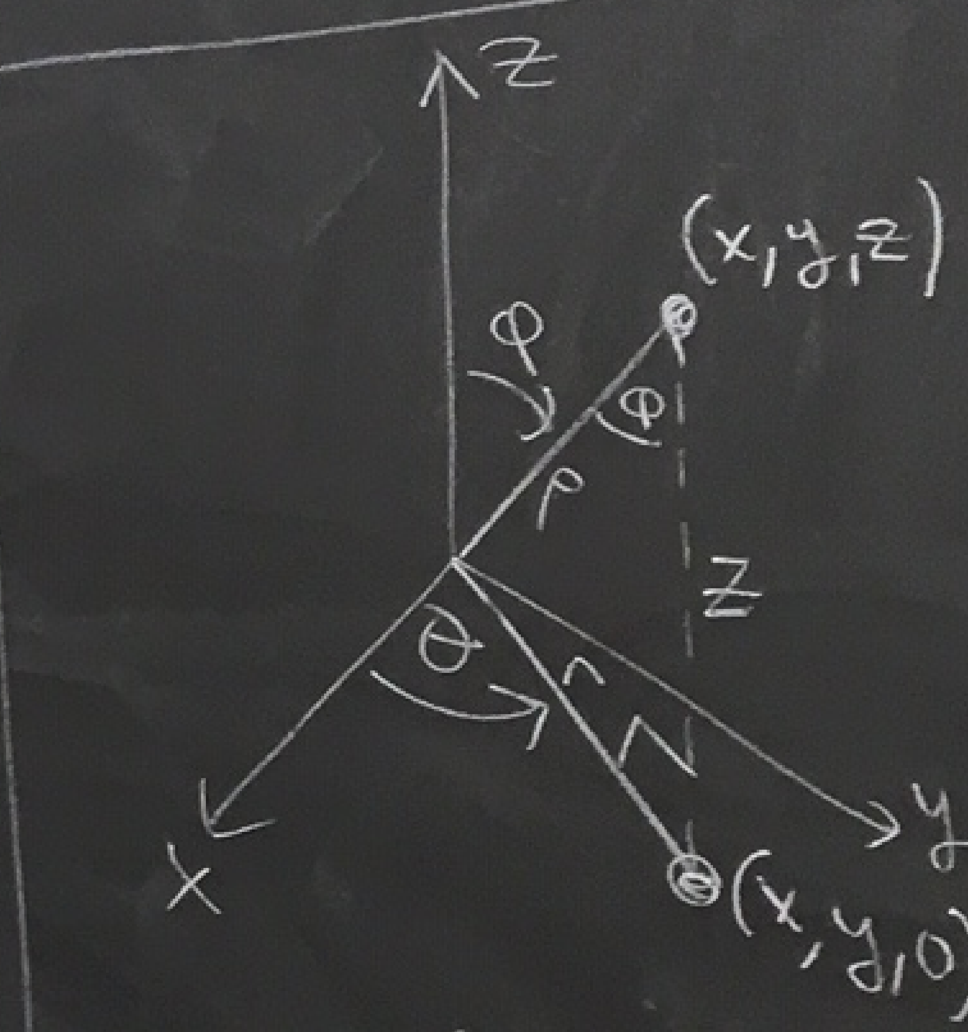
$$\rho=\cos(\varphi)$$

parameterize E

$$0\le\rho\le\cos(\varphi)$$

$$0\le\varphi\le\pi/4$$

$$0\le\theta\le 2\pi$$



$$\cos(\varphi)=\frac{z}{\rho}$$

$$z=\rho\cos(\varphi)$$

$$\text{Volume of } E = \iiint_E 1\, dV$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\cos(\varphi)} \underbrace{\rho^2 \sin(\varphi)\, d\rho\, d\varphi\, d\theta}_{dV} = \int_0^{2\pi}\int_0^{\pi/4} \frac{\rho^3}{3}\sin(\varphi)\Big|_{\rho=0}^{\cos(\varphi)} d\varphi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \frac{1}{3}\cos^3(\varphi)\sin(\varphi)\, d\varphi\, d\theta = \int_0^{2\pi}\int_1^{\frac{\sqrt{2}}{2}}\left(-\frac{1}{3}u^3\, du\right) d\theta = -\int_0^{2\pi} \frac{u^4}{12}\Big|_1^{\frac{\sqrt{2}}{2}} d\theta$$

$\varphi = 0 \to u = \cos(0) = 1$

$\varphi = \frac{\pi}{4} \to u = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$u = \cos(\varphi)$

$du = -\sin(\varphi)\, d\varphi$

$-du = \sin(\varphi)\, d\varphi$

$$= -\frac{1}{12}\int_0^{2\pi}\left(\left(\frac{\sqrt{2}}{2}\right)^4 - 1^4\right) d\theta = -\frac{1}{12}\int_0^{2\pi}\left(-\frac{3}{4}\right) d\theta = \frac{1}{16}\int_0^{2\pi} d\theta$$

$$= \frac{1}{16}\theta\Big|_0^{2\pi} = \frac{2\pi}{16} = \frac{\pi}{8} \approx \boxed{0.3926}$$

14.1 – Vector Fields

$\boxed{2d}$

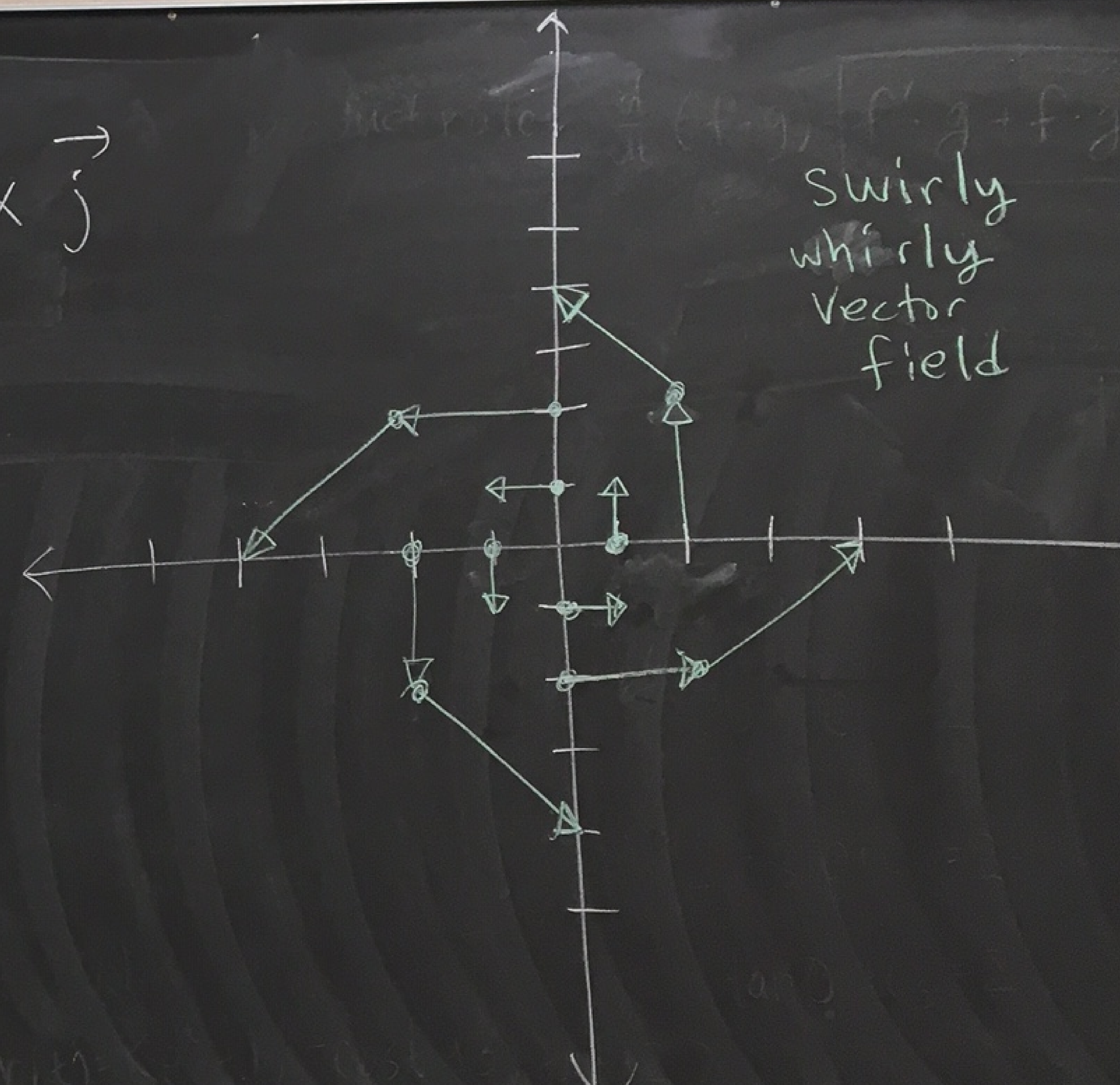
Def: Let D be a region in $\mathbb{R}^2$.
A vector field on D is a function
$\vec{F}$ that assigns to each point (x,y) in D
a two-dimensional vector $\vec{F}(x,y)$.

Ex: Consider the vector field given by $\vec{F}(x,y) = \langle -y, x \rangle = -y\,\vec{i} + x\,\vec{j}$

Sketch some vectors of $\vec{F}$.

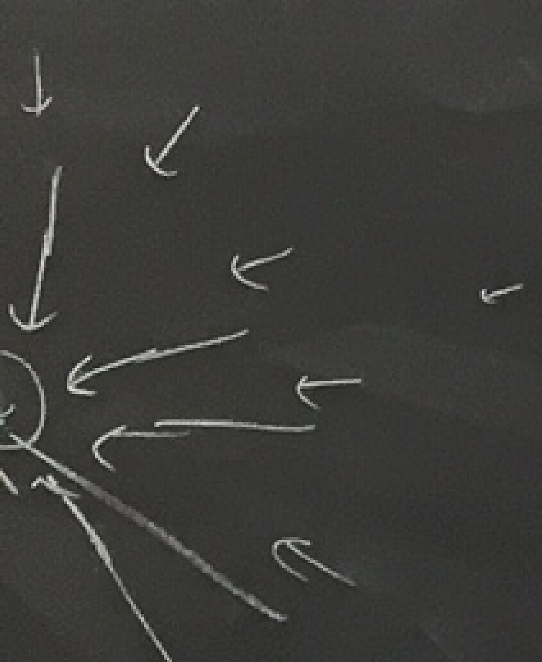
(x,y)	$\vec{F}(x,y)$
$(1,0)$	$\langle 0,1 \rangle$
$(0,1)$	$\langle -1,0 \rangle$
$(-1,0)$	$\langle 0,-1 \rangle$
$(0,-1)$	$\langle 1,0 \rangle$

(x,y)	$F(x,y)$
$(2,0)$	$\langle 0,2 \rangle$
$(-2,0)$	$\langle 0,-2 \rangle$
$(0,2)$	$\langle -2,0 \rangle$
$(0,-2)$	$\langle 2,0 \rangle$
$(2,2)$	$\langle -2,2 \rangle$
$(-2,-2)$	$\langle 2,-2 \rangle$



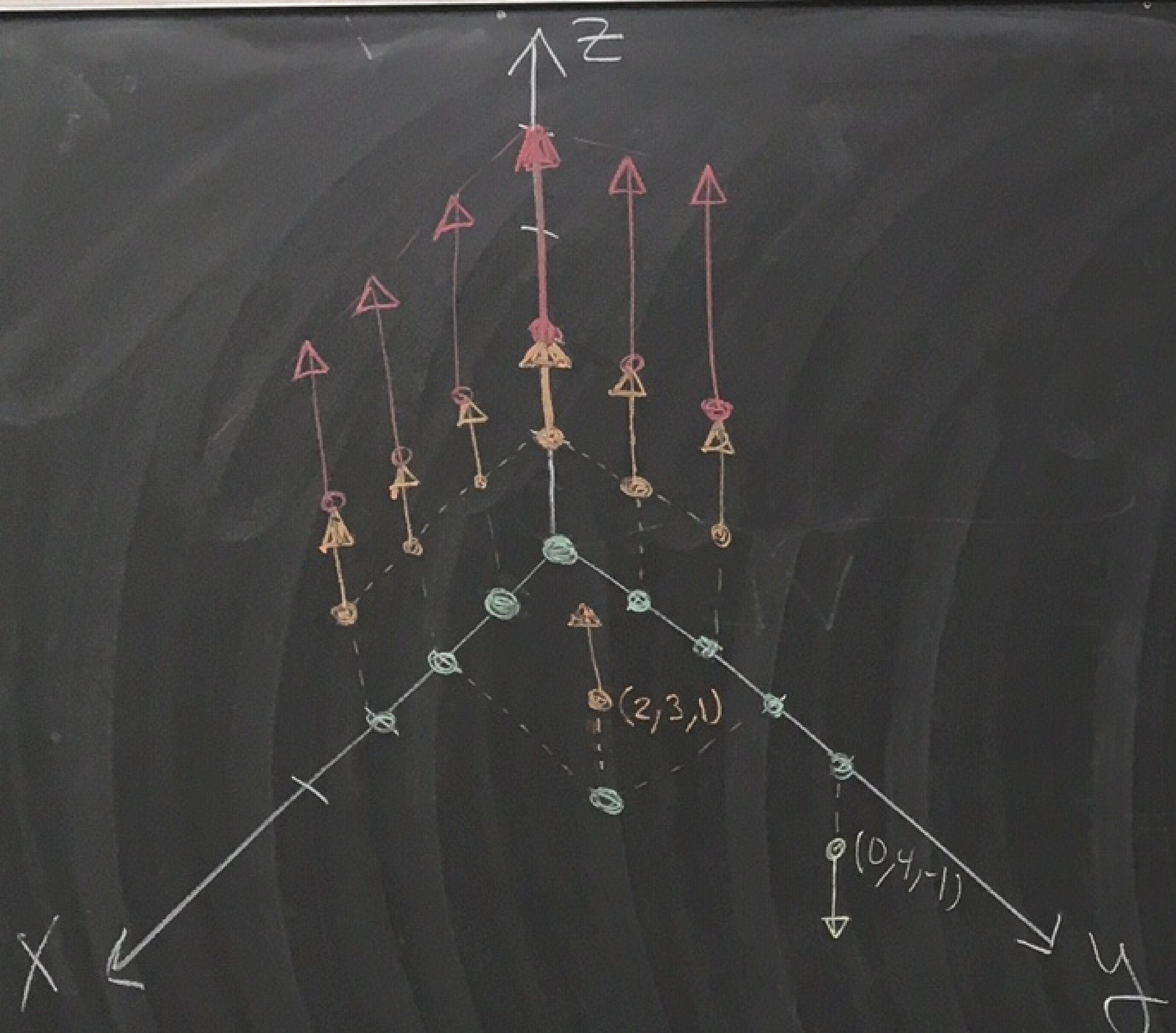
You can also make vector fields in $\mathbb{R}^3$:

<u>Ex:</u> Sketch some of the vector field given by $\vec{F}(x,y,z) = \langle 0,0,z \rangle$
$= z\vec{k}$



(x,y,z)	$\vec{F}(x,y,z) = \langle 0,0,z \rangle$
$(0,0,0)$	$\langle 0,0,0 \rangle$
$(0,0,1)$	$\langle 0,0,1 \rangle$
$(1,0,0)$	$\langle 0,0,0 \rangle$
$(0,1,1)$	$\langle 0,0,1 \rangle$
$(0,0,2)$	$\langle 0,0,2 \rangle$

X



A vector field $\vec{F}$ is
called a gradient vector field
if there exists a scalar
function f where $\nabla f = \vec{F}$.
If this is the case then we
say that $\vec{F}$ is a conservative vector field
and f is called a potential function
of $\vec{F}$.

Ex: $\vec{F}(x,y) = \langle 2xy,\ x^2 - 3y^2 \rangle$

is a conservative vector field.

Because $f(x,y) = x^2 y - y^3$

then $\nabla f = \langle 2xy,\ x^2 - 3y^2 \rangle = \vec{F}$

So, f is a potential function for $\vec{F}$.