Math 2150-01
5/5/25

On test 2 I wrote

GSF = your grade so far

This is your grade if you don't take the final

Final = test 1 + test 2

Use those tests and study guides to study.

The final can replace one or both tests

Final = Weds, 5/14, 2:30-4:30

I will put test 2 & solutions on the website tonight

Today = Laplace transform (not on final)

Weds = I'll be in class for any questions if you're taking the final

Ex: Solve

$$y'' + 4y = 5e^{-x}$$

$$y(0) = 2, \; y'(0) = 3$$

using the Laplace transform

Apply $\mathcal{L}$ to the equation

$$\mathcal{L}[y'' + 4y] = \mathcal{L}[5e^{-x}]$$

So,

$$\mathcal{L}[y''] + 4\mathcal{L}[y] = 5\mathcal{L}[e^{-x}]$$

Then,

$$\underbrace{\left(t^2\mathcal{L}[y] - ty(0) - y'(0)\right)}_{\mathcal{L}[y'']} + 4\mathcal{L}[y] = 5\mathcal{L}[e^{-x}]$$

Since $y(0)=2$ and $y'(0)=3$ we get

$$t^2\mathcal{L}[y]-2t-3+4\mathcal{L}[y]=5\mathcal{L}[e^{-x}]$$

Since $\mathcal{L}[e^{ax}]=\frac{1}{t-a}$ we have

$$(t^2+4)\mathcal{L}[y]-2t-3=\frac{5}{t+1}$$

Thus,

$$\mathcal{L}[y]=\frac{5}{(t+1)(t^2+4)}+\frac{2t}{t^2+4}+\frac{3}{t^2+4}$$

Need to do partial fractions on first term.

$$\frac{5}{(t+1)(t^2+4)}=\frac{At+B}{t^2+4}+\frac{C}{t+1}$$

So,

$$5=(At+B)(t+1)+C(t^2+4)$$

Plug different t's in.

$t=-1$: $5 = 0 + C(5) \leftarrow \boxed{C=1}$

$t=0$: $5 = (0+B)(0+1) + (1)(0^2+4)$

$5 = B + 4$

$\boxed{B=1}$

$t=1$: $5 = (A+1)(1+1) + (1)(1^2+4)$

$5 = 2A + 2 + 5$

$\boxed{A=-1}$

So,

$$\frac{5}{(t+1)(t^2+4)} = \frac{-t+1}{t^2+4} + \frac{1}{t+1}$$

So,

$$\mathcal{L}[y] = \frac{5}{(t+1)(t^2+4)} + \frac{2t}{t^2+4} + \frac{3}{t^2+4}$$

becomes

$$\mathcal{L}[y] = \underbrace{\left(\frac{-t+1}{t^2+4} + \frac{1}{t+1}\right)} + \frac{2t}{t^2+4} + \frac{3}{t^2+4}$$

Then,

$$\mathcal{L}[y] = \frac{t}{t^2+4} + \frac{4}{t^2+4} + \frac{1}{t+1}$$

So,

$$\mathcal{L}[y] = \frac{t}{t^2+4} + 2\left(\frac{2}{t^2+4}\right) + \frac{1}{t+1}$$

What y satisfies the above?

$$\boxed{y = \cos(2x) + 2\sin(2x) + e^{-x}}$$

$\mathcal{L}[\cos(kx)] = \frac{t}{t^2+k^2}$ | $\mathcal{L}[\sin(kx)] = \frac{k}{t^2+k^2}$

$$\boxed{\mathcal{L}[e^{ax}] = \frac{1}{t-a}}$$

So,

$$y = \underbrace{\cos(2x) + 2\sin(2x)}_{y_h \text{ term}} + \underbrace{e^{-x}}_{y_p \text{ term}}$$

Solves

$$y'' + 4y = 5e^{-x}, \quad y(0) = 2, \; y'(0) = 3$$