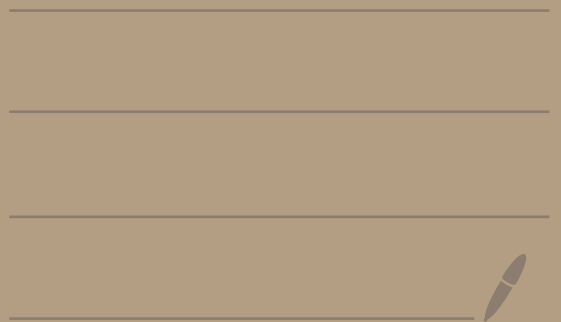


Math 2150-02

5/5/25



On test 2 I wrote

GSF = your grade so far

This is your grade if you don't take the final

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Final = test 1 + test 2

Use those tests and study guides to study.

The final can replace one or both tests

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Final = Weds, 5/14, 12-2

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I will put test 2 & solutions on the website tonight

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Today = Laplace transform (not on final)

Weds = I'll be in class for any questions if you're taking the final

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Ex: Solve

$$y'' + 4y = 5e^{-x}$$

$$y(0) = 2, \quad y'(0) = 3$$


Using the Laplace transform

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Suppose the Laplace transform exists for the initial value problem above.

Apply  $\mathcal{L}$  to the above equation.

$$\mathcal{L}[y'' + 4y] = \mathcal{L}[5e^{-x}]$$


$$\mathcal{L}[y''] + 4\mathcal{L}[y] = 5\mathcal{L}[e^{-x}]$$

$$(\text{using } \mathcal{L}[c_1 f + c_2 g] = c_1 \mathcal{L}[f] + c_2 \mathcal{L}[g])$$

We get:

$$\underbrace{(t^2 \mathcal{L}[y] - t y(0) - y'(0))}_{\mathcal{L}[y'']} + 4 \mathcal{L}[y] = 5 \mathcal{L}[e^{-x}]$$

Since  $y(0)=2$ ,  $y'(0)=3$  we get

$$t^2 \mathcal{L}[y] - 2t - 3 + 4 \mathcal{L}[y] = 5 \mathcal{L}[e^{-x}]$$

Since  $\mathcal{L}[e^{-x}] = \frac{1}{t+1}$  we get

$$(t^2 + 4) \mathcal{L}[y] - 2t - 3 = 5 \left( \frac{1}{t+1} \right)$$

Thus,

$$\mathcal{L}[y] = \frac{1}{(t^2 + 4)} \left[ \frac{5}{t+1} + 2t + 3 \right]$$

So,

$$\mathcal{Z}[y] = \frac{5}{(t+1)(t^2+4)} + 2\left(\frac{t}{t^2+4}\right) + 3\left(\frac{1}{t^2+4}\right)$$

Need to do partial fractions  
on the term  $\frac{5}{(t+1)(t^2+4)}$ .

$$\frac{5}{(t+1)(t^2+4)} = \frac{At+B}{t^2+4} + \frac{C}{t+1}$$

$$5 = (At+B)(t+1) + C(t^2+4)$$

← plug various  $t$  in

$t = -1$ :  $5 = 0 + C(5) \leftarrow \boxed{C=1}$

$t = 0$ :  $5 = (B)(1) + \underbrace{(1)}_C(4) \leftarrow \boxed{B=1}$

$t = 1$ :  $5 = \underbrace{(A+1)}_{A+B}(2) + \underbrace{(1)}_C(5) \leftarrow \boxed{A=-1}$

$$\text{So, } \frac{5}{(x+1)(x^2+4)} = \frac{-x+1}{x^2+4} + \frac{1}{x+1}$$

Thus,

$$\mathcal{L}[y] = \frac{5}{(x^2+4)(x+1)} + 2 \left( \frac{x}{x^2+4} \right) + 3 \left( \frac{1}{x^2+4} \right)$$

$$\mathcal{L}[y] = \left( \frac{-x}{x^2+4} + \frac{1}{x^2+4} + \frac{1}{x+1} \right) + 2 \left( \frac{x}{x^2+4} \right) + 3 \left( \frac{1}{x^2+4} \right)$$

So,

$$\mathcal{L}[y] = \frac{x}{x^2+4} + 2 \left( \frac{2}{x^2+4} \right) + \frac{1}{x+1}$$

LAST TIME

$$\mathcal{L}[\sin(kx)] = \frac{k}{x^2+k^2}$$

$$\mathcal{L}[e^{ax}] = \frac{1}{x-a}$$

$$\mathcal{L}[\cos(kx)] = \frac{x}{x^2+k^2}$$

So,

$$y = \cos(2t) + 2\sin(2t) + e^{-x}$$

$y_h$

$y_p$