Math 2150-02
5/5/25

On test 2 I wrote
GSF = your grade so far
This is your grade if you don't take the final

Final = test 1 + test 2
Use those tests and study guides to study.
The final can replace one or both tests

Final = Weds, 5/14, 12-2

I will put test 2 & solutions on the website tonight

Today = Laplace transform (not on final)
Weds = I'll be in class for any questions if you're talking the final

Ex: Solve

$$y'' + 4y = 5e^{-x}$$
$$y(0) = 2, \quad y'(0) = 3$$

Using the Laplace transform

Suppose the Laplace transform exists for the initial value problem above.

Apply $\mathcal{L}$ to the above equation.

$$\mathcal{L}[y'' + 4y] = \mathcal{L}[5e^{-x}]$$

$$\mathcal{L}[y''] + 4\mathcal{L}[y] = 5\mathcal{L}[e^{-x}]$$

(using $\mathcal{L}[c_1 f + c_2 g] = c_1\mathcal{L}[f] + c_2\mathcal{L}[g]$)

We get:

$$\underbrace{\left(t^2\mathcal{L}[y]-t\,y(0)-y'(0)\right)}_{\mathcal{L}[y'']}+4\mathcal{L}[y]=5\mathcal{L}[e^{-x}]$$

Since $y(0)=2$, $y'(0)=3$ we get

$$t^2\mathcal{L}[y]-2t-3+4\mathcal{L}[y]=5\mathcal{L}[e^{-x}]$$

Since $\mathcal{L}[e^{-x}]=\frac{1}{t+1}$ we get

$$(t^2+4)\mathcal{L}[y]-2t-3=5\left(\frac{1}{t+1}\right)$$

Thus,

$$\mathcal{L}[y]=\frac{1}{(t^2+4)}\left[\frac{5}{t+1}+2t+3\right]$$

So,

$$\mathcal{L}[y] = \frac{5}{(t+1)(t^2+4)} + 2\left(\frac{t}{t^2+4}\right) + 3\left(\frac{1}{t^2+4}\right)$$

Need to do partial fractions on the term $\frac{5}{(t+1)(t^2+4)}$.

$$\frac{5}{(t+1)(t^2+4)} = \frac{At+B}{t^2+4} + \frac{C}{t+1}$$

$$\boxed{5 = (At+B)(t+1) + C(t^2+4)} \leftarrow$$ plug various t in

$\underline{t=-1:}$ $5 = 0 + C(5) \leftarrow \boxed{C=1}$

$\underline{t=0:}$ $5 = (B)(1) + \underbrace{(1)}_{C}(4) \leftarrow \boxed{B=1}$

$\underline{t=1:}$ $5 = \underbrace{(A+1)}_{A+B}(2) + \underbrace{(1)}_{C}(5) \leftarrow \boxed{A=-1}$

So, $\dfrac{5}{(t+1)(t^2+4)} = \dfrac{-t+1}{t^2+4} + \dfrac{1}{t+1}$

Thus,

$$\mathcal{L}[y] = \underbrace{\frac{5}{(t^2+4)(t+1)}} + 2\left(\frac{t}{t^2+4}\right) + 3\left(\frac{1}{t^2+4}\right)$$

$$\mathcal{L}[y] = \overbrace{\left(\frac{-t}{t^2+4} + \frac{1}{t^2+4} + \frac{1}{t+1}\right)} + 2\left(\frac{t}{t^2+4}\right) + 3\left(\frac{1}{t^2+4}\right)$$

So,

$$\mathcal{L}[y] = \frac{t}{t^2+4} + 2\left(\frac{2}{t^2+4}\right) + \frac{1}{t+1}$$

LAST TIME

$$\mathcal{L}[\sin(kx)] = \frac{k}{t^2+k^2}$$

$$\mathcal{L}[e^{ax}] = \frac{1}{t-a} \qquad \mathcal{L}[\cos(kx)] = \frac{t}{t^2+k^2}$$

So,

$$\boxed{y = \underbrace{\cos(2t) + 2\sin(2t)}_{y_h} + \underbrace{e^{-x}}_{y_p}}$$