Math 2150 - Test 2 - Spring 2025

Name:____

Directions:

Show steps for full credit.
Also so I can give you partial credit if needed.
You may use a calculator on this test that isn't your phone or isn't internet enabled.

Score			
1		2	
3		4	
5			
Total			

Separable	$f(x) = g(y)\dfrac{dy}{dx}$	Separate $\dfrac{dy}{dx}$ and then integrate
Linear	$y' + a(x)y = b(x)$	Multiply by $e^{A(x)}$ where $A(x) = \int a(x)dx$
Exact	$M(x,y) + N(x,y) \cdot y' = 0$	Test: $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ Find f where $\dfrac{\partial f}{\partial x} = M$ and $\dfrac{\partial f}{\partial y} = N$ Solution: $f(x,y) = c$
Constant coefficient	$a_2 y'' + a_1 y' + a_0 y = 0$	Two real roots r_1, r_2 use $e^{r_1 x}$ and $e^{r_2 x}$ If double real root r use e^{rx} and xe^{rx} If complex roots $r = \alpha \pm \beta i$ use $e^{\alpha x}\cos(\beta x)$ and $e^{\alpha x}\sin(\beta x)$
Variation of parameters	$y'' + a_1(x)y' + a_0(x)y = b(x)$ y_1 and y_2 sols. to homogeneous eqn.	$y_p = v_1 y_1 + v_2 y_2$ $v_1 = \int \dfrac{-y_2 \cdot b(x)}{W(y_1, y_2)} dx$ $\qquad v_2 = \int \dfrac{y_1 \cdot b(x)}{W(y_1, y_2)} dx$
Reduction of Order	$y'' + a_1(x)y' + a_0(x)y = 0$ on the interval I	y_1 is a solution that isn't zero on I $y_2 = y_1 \cdot \int \dfrac{e^{-\int a_1(x)dx}}{y_1^2} dx$
Euler's method	$y' = f(x,y)$ $y(x_0) = y_0$	$x_n = x_{n-1} + h$ $y_n = y_{n-1} + h \cdot f(x_{n-1}, y_{n-1})$

$b(x)$	y_p guess for undetermined coefficients
constant	A
$5x - 3$	$Ax + B$
$10x^2 - x + 1$	$Ax^2 + Bx + C$
$\sin(6x)$	$A\cos(6x) + B\sin(6x)$
$\cos(6x)$	$A\cos(6x) + B\sin(6x)$
e^{3x}	Ae^{3x}
$(2x+1)e^{3x}$	$(Ax + B)e^{3x}$
$x^2 e^{3x}$	$(Ax^2 + Bx + C)e^{3x}$
$e^{3x}\sin(4x)$	$Ae^{3x}\cos(4x) + Be^{3x}\sin(4x)$
$e^{3x}\cos(4x)$	$Ae^{3x}\cos(4x) + Be^{3x}\sin(4x)$

Taylor series: $f(x_0) + f'(x_0)(x - x_0) + \dfrac{f''(x_0)}{2!}(x - x_0)^2 + \dfrac{f^{(3)}(x_0)}{3!}(x - x_0)^3 + \dfrac{f^{(4)}(x_0)}{4!}(x - x_0)^4 + \dots$

$\int u\,dv = uv - \int v\,du$ $\qquad \int \sin(x)dx = -\cos(x)$ $\qquad \int \cos(x)dx = \sin(x)$ $\qquad W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

1. [10 points]
One can show that the solution to $y'' - 2y' - 3y = 0$ is $y_h = c_1 e^{3x} + c_2 e^{-x}$.

(a) Use undetermined coefficients to find a particular solution y_p to

$$y'' - 2y' - 3y = 9x^2 + 1$$

(b) State the general solution to $y'' - 2y' - 3y = 9x^2 + 1$.

2. [10 points] Consider the ODE

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

One can show that the solution to the homogeneous equation is given by

$$y_h = c_1 e^{2x} + c_2 x e^{2x}$$

Use variation of parameters to find a particular solution y_p to

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

3. [10 points] Suppose you are given that a solution to

$$x^2y'' + 2xy' - 6y = 0$$

is $y_1 = x^2$ on $I = (0, \infty)$

(a) Find another solution y_2 that is linearly independent to y_1.

(b) State the general solution to $x^2y'' + 2xy' - 6y = 0$ on I.

4. [10 points] Find a power series expansion for $f(x) = x^4$ centered at $x_0 = 2$. State the radius of convergence r for the series.

5. [10 points] (a) Find a power series solution to

$$y'' - xy' + y = 0, \qquad y'(0) = 1, \;\; y(0) = 1$$

In your answer go out to the 4-th power of x in the series.

(b) For what values of x does the power series converge? That is, what is it's radius of convergence r?

(Extra page if you need it.)