Math 4460 - Test 2 - Spring 2025

Name:_____

Testing rules:

You may use a calculator that is not your phone. It cannot be internet-enabled.

Score			
1		2	
3		A or B	
C or D		Total	

1. [15 points - 5 each]

(a) True or False? Justify your answer.

$$-5 \equiv 15 \pmod{4}$$

(b) True or False? Justify your answer.

$$\overline{4} = \overline{100}$$
 in \mathbb{Z}_5

(c) Calculate in \mathbb{Z}_6 and reduce your answer \overline{x} so that $0 \leq x \leq 5$

$$\overline{-2} \cdot \overline{5} + \overline{2}^4 + \overline{2} \cdot \overline{4} \cdot \overline{10}$$

2. [10 points]

- (a) List the elements of \mathbb{Z}_{10}^{\times} .
- (b) Calculate $\overline{3}^{1002}$ in \mathbb{Z}_{10}^{\times} .

- **3.** [10 points] Consider $\mathbb{Z}_7^{\times} = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}\}.$
 - (a) Find the inverse of $\overline{3}$ in \mathbb{Z}_7^{\times} .
 - (b) Find one primitive root in \mathbb{Z}_7^{\times} .

(A or B) [10 points] Pick ONE of problems A or B

Only do one. If you turn in two then I will grade A.

A) Prove that $\sqrt[5]{5}$ is irrational.

B) Let p be a prime and let m and n be positive integers. Let $\overline{a} \in \mathbb{Z}_p^{\times}$. Prove that if $m \equiv n \pmod{p-1}$, then $\overline{a}^m = \overline{a}^n$ in \mathbb{Z}_p^{\times} .

(C or D) [10 points] Pick ONE of problems C or D

Only turn in one. If you turn in both then I will grade C.

C) Let p be an odd prime and x be an integer. Prove: If $x^2 + x \equiv 2 \pmod{p}$, then $x \equiv 1 \pmod{p}$ or $x \equiv -2 \pmod{p}$.

D) Prove that $15x^2 - 10x + 7y^2 = 11$ has no integer solutions.